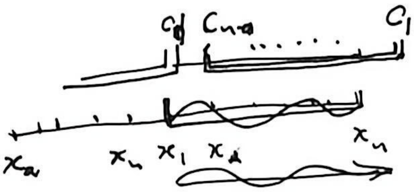
Circulation mat.

$= \wedge$

$c_1 \cdot F(\overset{\|}{FC})F'$ — 第一行



(代数也相似)

AB & BA 非零λ相同.

$\|I\|=1$, $\|O\|<1$,

$\sum_k^\infty A^k$ (i.e. $I+O$ nonsing) $\|\cdot\| \leq \frac{1}{1-\|O\|}$.

$\|\ \|_\varepsilon \leq \rho+\varepsilon > 0$.

$C\|\ \|_\varepsilon$.

$\|\ \|_2 \leq \|\ \|_F \leq n\|\ \|_2$.

Pagerank 扩散

Adj. L. λ^{-1}

□ modeling: Convolution 图; ode pde (模型问题) ($F \wedge F'$)

(回顾: $\|\cdot\|$, ρ,) PF Thm. MP^+.

0. □ solve by (det) Cramer

什么问题? 怎么研究? —— 分解 { co (Chol.) QR unique when A nonsing, m=n. Spec < Schur, diag. Jordan. $U\Sigma V'$. $\min\|A-Z\|_{2/F}$.

($\mathbb{E}^3$ $\leq u$). 敏感度 (back误差 κ_f * □ $|\frac{xf'}{f}|$. 例). 复杂度 (计算)

□ $fl(x'y)$. 高性能 □ LU 存储. BLAS. LAPACK.

1. inv. $Ax=b$. Gauss 消去 i.e. LU分解 $L_k = I - l_k e_k'$, $L = I + l_1e_1' + \cdots + l_{n-1}e_{n-1}'$. PA QR问题

(Schur补 $\frac{2}{3}n^3$) □ LU decomp. — Partial pivoting. $+O(n^2)$

(用A下半记L, $p=[1,\cdots,n]$ 记P) Pivoting (顺主子式 nonsing.) △ inv. A, $PA=LU$. □ 列主 LU decp

Cholesky (对 i.e. $A_{22} - l_k l_k'$ $\frac{1}{3}n^3$) $L_{n-1}P_{n-1}\cdots L_1P_1$ let $L_k' = (P_{n-1}\cdots P_{k+1})L_k(P_{k+1}'\cdots P_{n-1}')$,

$= \underbrace{L_{n-1}'\cdots L_1'}_{L^{-1}}P$. $= I - (P_{n-1}\cdots P_{k+1})l_k e_k'$.

□ Chol. decomp. $l_k = \frac{a_k}{\sqrt{a_{kk}}}$. □ 数值稳定性 $\begin{pmatrix}10^{-4} & 1\\ 1 & 1\end{pmatrix}$ in Algo. $A(k,k:n) \leftrightarrow A(j,k:n)$

(if S nonsing, (LDLT) $PAP' = LDL'$ with D 1x1 or 2x2 block.) 误: (元素乘X) $|\delta A| \lesssim (n+1)u|\hat{L}||\hat{L}'|$ ($\rho \leq 2$) 对角占优 对称正定 } 时稳定. (see Chol.)

LDL' 问题. by $l_{ii} = \sqrt{a_{ii} - \sum_k^{i-1} l_{ik}^2}$ i.e. $fl(\ldots) = \hat{l}_{ii}$

$(A+\delta A)\hat{x} = b$. $l_{ij} = \frac{1}{l_{jj}}(a_{ij} - \sum_k^{j-1} l_{ik}l_{jk})$ e.g.) □ $\|\delta A\|_2 \leq n\|\delta A\|_2 \leq (3n+1)u$. $\frac{\|A\|_2}{1-n(n+1)u}$

$\underbrace{(3n+1)}_{n\ n\ n+1}u|\hat{L}||\hat{L}'|$. △ Gauss $3nu|\hat{L}||\hat{U}|$. (inv. 正对角 $\rho \leq 2$ when 列主 Gauss)

$\Rightarrow \frac{\|\delta A\|_\infty}{\|A\|_\infty} \leq 3n^3\rho u$. $\rho := \frac{\|U\|_m}{\|A\|_m}$.

(前误) 误: $\frac{\kappa(A)}{1-\kappa(A)\frac{\|\delta A\|}{\|A\|}}$ ($\rho \leq 2^{n-1}$ but actually smaller.)

(κ 病态解释)

(后误) $E = \frac{\tilde{r}\tilde{x}'}{\|\tilde{x}\|_2^2}$. 按列/行 △(多步*) i.e. $\langle q_k, q_i\rangle / \langle r_k, q_i\rangle$ mod. reortho. (CGS/MGS/CGS2) (用 v_k 记 H)

col. inv. min $\|A_{m\times n}x - b\|_2$ 2. GS 正交化 □ $O(4m\sum_1^{n-1}) = O(2mn^2)$. $H = I - 2\frac{1}{v'v}vv'$.

$v = x - (\pm\|x\|_2)e_1$ (可 $x + \mathrm{sign}(x_1)\|x\|e_1$ or $x - \|x\|e_1$ for 正 R) 数值良 $O(u)$

Householder □ $O(4\sum_{m-n+1}^{m} *^n) = O(2mn^2 - \frac{2}{3}n^3)$ but $x_1 - \|x\|_2 = \frac{-\sum_2^n x_i^2}{x_1 + \|x\|_2}$ 稳定)

Givens. $G(i,j;\theta)$. esp. $O(\frac{4}{3}n^3)$ (note Householder □ 是 n^3 复杂度

est. rank {SVD 列QR $\|A(k:n,j)\|_2 \max \approx 0$. $\boxed{r}$ $AP = Q\begin{pmatrix}R_1 & *\\ 0 & 0\end{pmatrix}$ 算 Ax 而非 矩阵* 的 n^3 正则化 $\left\|\begin{pmatrix}A\\ \sqrt{\varepsilon}I\end{pmatrix}x - \begin{pmatrix}b\\0\end{pmatrix}\right\|$

Normal eq. $\kappa_2(A)^2 = \kappa_2(A'A)$. (note by $\|A\|_2^2 = \|A'A\|_2$) 线性约束 s.t. $Bx=d$. B' $R'y=d$ $x = Q\begin{pmatrix}y\\z\end{pmatrix}$

{ NG NC QR ✓ (不变 κ) $\to$ min $= \|(I-QQ')b\|$ i.e. κ 的投影 to $Col(A)$ (紧 QR)

3. (实) Schur decomp. Spec decomp. (Pf by 归纳) GF Thm. Weyl Thm. (Pf by C-F) (W-H Thm.)

(交分表示) Gerschgorin. $\exists$ eigenpair (λ_1, x) : λ 变化 1.2.

幂法 $\lambda_1^k(q_1 + \alpha_2(\frac{\lambda_2}{\lambda_1})^k q_2 + \cdots)$ $w \overset{\text{算}}{=} Ax^{(k)}$, $x^{(k+1)} = w^{(k)}/\|w^{(k)}\|_{2/\infty\cdots}$

(收敛: $\sin^2\theta_k$, $|\lambda^{(k)} - \lambda_1|$ at $O((\frac{\lambda_2}{\lambda_1})^{2k})$ end) $|\frac{\lambda_2}{\lambda_1}|$, $\pm\lambda_1$ 等多值 ×, (use $\|\cdot\|_2$ upd)

(□ $\|\cdot\|\cdot\|\cdots$ 或不使用 Rayleigh 而是 max 位估计 λ □ $/\|\cdot\|_\infty\cdots$)

反幂 对 $(A-\mu I)^{-1}$ 应用 power, 可预 LU decomp. 条件: 同 λ_1 需半单 (几重 代重), 有 q_1 上投影.

(常用于) 一次迭代法 for eigenvec. (i.e. $A - \hat{\lambda}_i I$ 病态性 不影响) Rayleigh 商迭 $(A - \lambda^{(k)} I) w^{(k)} = x^{(k)}$

local 三次敛, but 复杂度高 ($A - \lambda^{(k)} I$ 均不同)

$\|E^{(k)}\| \cdot \frac{|\lambda_{n}|}{|\lambda_{n-1}|}$ q_n main

$|\lambda_1| > \cdots > |\lambda_n| > 0$

△ 且 左 eigenvecs 有 LU decomp. 正交迭代 → QR 法

ℓ_{m} 与 $\lambda^{(0)}, x^{(0)}$ 有关, 收敛到 ... 排列阵 e.g.

$Z_k = AQ_{k-1}$ $T_{k-1} = Q_k R_k$ (intuition: $\rho(x) \overset{\triangle}{=} \frac{x'Ax}{x'x} = \arg\min_\alpha \|Ax - \alpha x\|$,

T_k 下半 → 0, diag → $|\lambda_i|$. $Q_k R_k = Z_k$ $T_k = R_k Q_k$ i.e. $T_k = \tilde{Q}_k' A \tilde{Q}_k$ also shown $\rho(q_i) = \lambda_i$ and $\nabla\rho(q_i) = 0$.

(分块) $Q^{(k)}_{n\times r} \to Q_{\text{前}r\text{列}}$ with velo $\frac{|\lambda_{r+1}|}{|\lambda_r|}$, $\max_{1\le r\le n} \frac{|\lambda_{r+1}|}{|\lambda_r|}$ in whole. $\tilde{Q}_k \overset{\triangle}{=} Q_1 \cdots Q_k$ in 正迭 (先 后~) (thus $\rho(x) \xrightarrow{est} \rho(q)$)

$T_0 = Q_0' A Q_0$:

Tridiag 经 QR step 仍为 tridiag. 关系: $A^k = \tilde{Q}_k \tilde{R}_k$. $A^{-k'}P = \tilde{Q}_k P \cdot P\tilde{R}_k^{-T} P$. QR: $O(\varepsilon) \to O(\varepsilon^3)$, $\varepsilon \frac{b}{a}$

(or 上 Hessenberg) (Pf by Givens) (e_1 用 power 法 得 $\tilde{q}_1$). (反 power) e_n 得 $\tilde{q}_n$ □ $\begin{pmatrix} a & \varepsilon \\ \varepsilon & b \end{pmatrix}$ W: $\to 0$ $\frac{\varepsilon^3}{(b-a)^2}$

Rayleigh 商位移: also 有 $\Pi(T_0 - \mu_k I) = \tilde{Q}_k \tilde{R}_k$, ~ Wilkson: $\begin{pmatrix} \alpha_{n-1} & \beta_{n-1} \\ \beta_{n-1} & \alpha_n \end{pmatrix}$'s (选靠近 α_n 的)

选 $\mu_k = \tilde{q}_{n(k-1)}' T_0 \tilde{q}_{n(k-1)} = T_{k-1}(n,n)$. 同, 三次敛.

$O(\frac{4}{3}n^3 / 9n^3) + O(n^2)$ (对角化后的 step per) (for T) T/H 化 基本

△ 隐式 Q Thm.

第一列 G_1. bulgechasing.

完整算法 □ 1||2 sym. (找划分、判零) Francis M 三... double-step. (Wilkson 更好) e.g.

$\|A\|_2 u$. u. AED

图1 | 图2 sym.

Jacobi、二分、分治 glue.

step. off(A) ≤ c‖A‖$_F$. 并行.

△ 当 $|\tan\theta| \le 1$ 时. 某排列 $1 + \rho u'(D-\lambda I)^{-1} u$

选 e_i $A^{(k)} = \operatorname{diag}(\lambda_i's)$ ($O(n^2)$ eigenvec & val)

$(D-\lambda I)^{-1}u$. ← where $u = \begin{pmatrix} Q_1 \text{末列} \\ Q_2 \text{首列} \end{pmatrix}$

a sweep $N = \frac{n(n-1)}{2}$, each $O(6n)$. 稳定性 subtle

(esp. 过关扫描) (e.g. $\lambda_i \approx \lambda_j$) $\begin{pmatrix} Q_1 & \\ & Q_2 \end{pmatrix}\left(\begin{pmatrix} \Lambda_1 & \\ & \Lambda_2 \end{pmatrix} + \rho u u'\right)\begin{pmatrix} Q_1' & \\ & Q_2' \end{pmatrix}$

e_i 二次敛. (至少线性, $(1-\frac{1}{N})^k$) 由 $\text{off}^2 - 2a_{ij}^2$ 知 $\begin{pmatrix} T_1 & \rho \\ \rho & T_2 \end{pmatrix}$

并行.

敏: $|\Delta\lambda| \lesssim \frac{\|\Delta A\|_2}{|y'x|}$ ($Ax = \lambda x$, $y'A = \lambda y'$ with $\|x\| = \|y\| = 1$) esp. sym A. $|\Delta\lambda| \lesssim \|\Delta A\|_2$ and ΔA sym 时 i.e. Weyl Thm.

$\{p_{ii}^t > 0\}$ gcd $\overset{?}{=} 1$ 周期性

λ_i (λ_{r+1}) 分离程度 $\sin 2\theta \le \frac{4\|\delta\|_2}{d}$ ← 刻画. D-K $\sin\Theta$ Thm. (for 多 eigenval 状态.)

□ (左) 随阵 $P^k \pi_0$. 本原阵 Primitive. e.g. 吸收态不唯一 时. 例链 1, 2, 3 非次不可约: may $|\lambda| = \rho(A)$ ($\lambda \neq \rho(A)$) and its x 非负

P Thm. P-F Thm. Perron $\pi 1'$ proj. ←e_k $\frac{A^k}{\rho(A)^k}$ (非周期时~ or 正~). (λ's x 严格正)

S.W. Cesàro $\frac{1}{n}\sum_{k=0}^{n-1} P^k$ 正: not

(i.e. 幂法在 ≥ 0 irre / > 0 A 下收敛性) by Birkhoff.

凸构造 solving 非凸 optimize

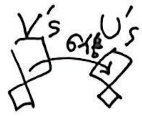
□ 图嵌入 $\min \operatorname{tr}(X'LX)$. s.t. $1'X=0$, $X'X=I$. by minimax, $X^* = [q_2, \cdots, q_{k+1}]$

$X \triangleq \begin{pmatrix} x_1' \\ \vdots \\ x_n' \end{pmatrix}$ $\quad (q_1 = \frac{1}{\sqrt{n}}$ for $\lambda_1 = 0)$

A. SVD existence (Pf by 归纳 取 $\sigma_1 = \|A\|_2$, $\exists A v_1 = \sigma_1 u_1$, show 对 $(u_1 U_1)' A (v_1 V_1) = \begin{pmatrix} \sigma_1 & 0 \\ 0 & B \end{pmatrix}$)

or def v_i's as $A'A$, show ... u_i's.

<核心> $\|\cdot\|_* = \sum_i \sigma_i$ 为核 (σ vec 的 ℓ_1 norm) 凸拓了也. Ky Fan-k norm, Schatten-p norm (Hölder ineq.)



V's 旋 U's

EYM Thm. for 酉不变 norm. (Pf by $\|\cdot\|_2$ 反证, $\|\cdot\|_F$ $|\Sigma - Z'|$). σ-变分表示, σ Weyl.

SVD (隐式B) 算法 $O(4mn^2 - \frac{8}{3}n^3)$. (bidiag化; 划分,降级; $P'BQ$)

Golub-Kahan.

& another view: $\exists$ 排列阵 P s.t. $P' \begin{pmatrix} & B' \\ B & \end{pmatrix} P$ = GK tridiag. 累乘

$\frac{1}{\sqrt{2}} \begin{pmatrix} v & v \\ u & -u \end{pmatrix}$.

□ 秩亏 LS. $x^* = \sum_{i=1}^{r} \frac{u_i' b}{\sigma_i} v_i$. ($A^+ = V\Sigma^{-1}U'$ if $\sigma_i \neq 0$)

(Pf 借助 四图) ridge $x_\tau^* = \sum_i \frac{\sigma_i}{\sigma_i^2 + \tau} (u_i' b) v_i$.

total LS $x^* = -\frac{v_{12}}{v_{22}}$. esp. Deming Reg.

$\min_{s.t.} \|[E, \delta]\|_F^2$ $[A+E, b+\delta]\begin{pmatrix} x^* \\ -1 \end{pmatrix} = 0$ (最小 $\sigma_{\min}$ 奇异向量 v of aug $[A, b]$) (x, y 同方差偏误时)

□ PCA. find $Q_{n\times p}$ i.e. $\bar{X}$ 奇异分解

① min 投影误差 $\underbrace{QQ'(x-\bar{x}) + \bar{x}}$. $\|(I - QQ')\bar{X}\|_F^2$.

② max 投影后方差 $\|QQ'\bar{X}\|_F^2$ or $\|Q'\bar{X}\|_F^2$.

i.e. Cov 特征分解 $= \bar{X}\bar{X}' = U\Sigma^2U'$

$\bar{X}_{n\times m} \triangleq (x_1, \cdots, x_m)$.

$\Rightarrow Q = U_p = [u_1, \cdots, u_p]$.

□ 非负秩 k 近似

(非unique $WD \cdot D^{-1}H$, $\min \frac{1}{2}\|A - WH\|_F^2$ or $D_{KL}(A\|WH)$.

若允许负则为 SVD, $U\Sigma \cdot V'$ or $U \cdot \Sigma V'$) s.t. $W, H \geq 0$.

Lemma. upd. < alternating 凸优化 in step, 交替 W, H. multiply $\odot \frac{\sigma}{\delta + \varepsilon}$

e.g. 单词-话题-文本. by $\frac{\partial L}{\partial H} = \delta - \sigma \geq 0$.

M.t. 求导
$f(X) \in \mathbb{R}$. $df = \operatorname{tr}\left(\frac{\partial f}{\partial X}' dX\right)$
e.g. $\|\cdot\|_F$, NN.
★ norms' deriv.
< Yuai >

稀疏性 $A * U$

4. 算子分裂, 定常迭代. Dom. ⇒ nonsing.

!

收: Lem. Thm. Thm. for J. GS. SOR. $\omega \in (0, 2)$ (Dom) $(0,1]$ (S_{++})

对一般 A 的 J 与 GS 收性无必然联系.

相容次序阵: e.g. 块三对角 with D_i 对角 ρ 联系: A co., (从模型问题 pf 推)

def by $I_1, \cdots I_k^i$ / $\alpha D^{-1}L + \frac{1}{\alpha}D^{-1}U \triangleq B(\alpha)$ eigenval 与 α 无关.

; I_{k-1} $(i,j) \neq 0$ I_{k+1} $(i,j) \neq 0$ ($A_D \neq 0$ i.e. $a_{ii} \neq 0$)

B^J 的 eigenval 成对 $\mu, -\mu$; if $\lambda \neq 0$ is B_ω^{SOR} eigenval, $(\lambda + \omega - 1)^2 = \lambda\omega^2\mu^2$ 确定 μ 为 B^J's; if μ is B^J's, ~ λ 为 B_ω^{SOR}'s.

逆推乃求 $\frac{\partial L}{\partial W} \frac{\partial}{\partial S}$ $dL = \operatorname{tr}\left(\frac{\partial L}{\partial S}' dS\right)$

后向 2dim 转置乘 $Wx \rightarrow$

line search SD $\alpha = \frac{r'r}{r'Ar}$. by $\min_\alpha f(x_k + \alpha r_k)$ / note $f(x) = \frac{1}{2}\|x - x^*\|_A^2 + f(x^*)$

$\Rightarrow \rho(B^{GS}) = \rho(B^J)^2$

$\omega^* = \frac{2}{1 + \sqrt{1 - \rho(B^J)^2}}$, $\rho_\omega^{SOR*} = \frac{1 - \sqrt{1-\rho^2}}{1 + \sqrt{1-\rho^2}}$.

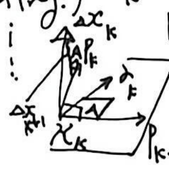
min (α, β) CG $p_k = r_k + \beta_{k-1}p_{k-1}$, $x_{k+1} = x_k + \alpha_k p_k$.

$\Delta x_{k+1} = \Delta x_k - \operatorname{Proj}_{A, r_k} \Delta x_k$
i.e. $\Delta x_{k+1} \perp_A r_k$, $r_{k+1}' r_k = 0$.

收: $\left(\frac{\kappa - 1}{\kappa + 1}\right)^k$, $2\left(\frac{\sqrt{\kappa}-1}{\sqrt{\kappa}+1}\right)^k$. $r_k = \pi_k(A) r_0$ then

(同SD 分析 $\frac{\partial}{\partial \alpha, \beta}$ / 几何 Proj. 得

$\alpha_k = \frac{r_k' p_k}{p_k' A p_k}$, $\beta_k = \frac{r_{k+1}' A p_k}{p_k' A p_k}$



$r_k' p_{k-1} = 0$. $r_k' r_{k-1} = 0$, $r_k' p_{k-2} = 0$, $p_k' A p_{k-1} = 0$.

$\underbrace{\perp, \perp_A}$ (Pf by 归纳)

$\Rightarrow \alpha = \frac{r'r}{p'Ap}$, $\beta = \frac{r_+' r_+}{r'r}$. Krylov $\mathcal{K}_{k+1}(A, r_0)$, $x_k \min x_0 + \mathcal{K}_k$

(pf $\Delta x_k \perp \mathcal{K}_{k-1}$, 由 $\|\cdot\|_A$ 的勾股得)

(Pf by $\|\Delta x_k\|_A = \|\pi_k(A)\Delta x_0\|_A$ and Cheb for κ. cf CG. or $\prod_k (I - \alpha_k A)$ of SD)

收速与 λ 分布有关.

《第1中》可定义 $R_\infty(B) = \lim_k R_k(B) \triangleq \lim_k -\frac{\ln\|B^k\|}{k} = -\ln\rho(B)$ 并在 B ∞ 时有一定关系(略).
接△.

for better $\kappa \approx 1$, PCG $M \approx A$, $C \triangleq \sqrt{M}$, $\tilde{x} = Cx$, $\tilde{r} = C^{-1}r$, $\tilde{p} = Cp$.

let $z_k \overset{一次}{=} M^{-1} r_k$, $\overset{一次}{A p_k}$ per step. $\underbrace{C^{-1}AC^{-1}}_{\tilde{A}}$. $Cx = C^{-1}b$.

then 直写 ~ 的 CG法 M^{-1} choose: …

$\alpha = \frac{r'z}{p'Ap}$, $\beta = \frac{r_+' z_+}{r'z}$, $p_+ = z_+ + \beta p$. $\perp_{M^{-1}}$, $\perp_A$. $\kappa(\tilde{A}) = \kappa(M^{-1}A)$.

扩展: α by …… β by $\frac{\nabla_+'\nabla_+}{\nabla'\nabla}$, $\frac{\nabla_+'(\nabla_+ - \nabla)}{\nabla'\nabla}$ or Krylov search.
or proximal, sub…

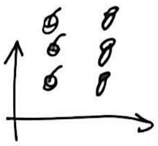
□ for (广义) eigen. $\min/\max\ \rho(x) = \frac{x'Ax}{x'Bx}$. $\nabla\rho(x) = \frac{2}{x'Bx}\cdot r$, ($r \triangleq Ax - \rho_{(x)}Bx$) SD. CG (loc. optimal) $Z_k = \mathrm{span}\{x_k, r_k, p_{k-1}\}$

(note $\rho(cx) = \rho(x)$) span$\{x_k, r_k\}$

□ 2维 Poisson. $n = O(N^2)$
故CG约 $O(N)$ step. (对于power法, $B = I$ 时,
$\cdot O(N^2)$/step. Span$\{r_k, x_k\} \ni Ax_k$ 作近似)

在 subsp. Z_k 上寻找 eigenvec 最佳近似.
(by min ρ)

□ clustering./谱图割.



min 割 / 归一化割

$\mathrm{cut}(A,\bar{A}) = \sum_{i\in A, j\in\bar{A}} w_{ij}$. $\frac{\mathrm{cut}}{v(A)} + \frac{\mathrm{cut}}{v(\bar{A})}$. $= \min\frac{x'Lx}{x'Dx}$

$\boxed{\text{Wishart. Hotelling } T \ldots \quad \mathrm{vec}(X).\ \text{Kronecker} \otimes}$

$x = \begin{cases} \frac{1}{v(A)} & i \in A \\ \frac{1}{v(\bar{A})} & i \in \bar{A} \end{cases}$ $\xrightarrow{\text{slack}}$ $x'D\mathbf{1} = 0$

i.e. $\min x'Lx$
s.t. $x'Dx = 1$ $\Rightarrow x^* = D^{-\frac{1}{2}} q_2$,
$x'De = 0$
q_2 of $D^{-\frac{1}{2}}LD^{-\frac{1}{2}}$'s.

sgn(x) 聚类. 采样 / 基底/字典

$b = \underbrace{\Phi\Psi}_{A_{m\times n},\ m<n} x^{n\times n}$. ($\min\|x\|_2$ 无法稀疏化)

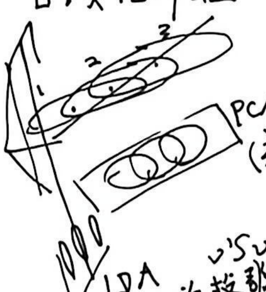
(esp. vol 按个数到

Fiedler $x'\mathbf{1} = 0$, $x'x = n$ or 1) (与求 q_2: 不同) 压缩感知.

□ 线性降维 { PCA 即奇异decomp. or $(Q = U_p)$ $\Sigma = XX'$ 特征decomp

LDA i.e. Fisher. $(Q_{c-1}$ of$)$ $S_w^{-1}S_b$ 特征decomp.

$\min\|x\|_0$ s.t. $b = Ax$.
slack ↓
$\min\|x\|_1$ s.t. ~. (△ K-sparse RIP. $\delta_k < \sqrt{2} - 1$).
slack ↓
l_1-正则化 $\min\frac{1}{2}\|Ax - b\|_2^2 + \tau\|x\|_1$



PCA.
(未使用,
多类信息)

$\max_{v, c-1} \frac{|v'S_b v|}{|v'S_w v|}$ between within.

by (acc.) PG. $\mathrm{Prox}_{\alpha_k h}(u) \triangleq \ldots$
$\Rightarrow$ 动量
$x_{k+1} = \mathrm{Prox}_{\alpha_k h}(y_k - \alpha_k\nabla f(y_k))$,
$y_{k+1} = x_{k+1} + \underline{\beta(x_{k+1} - x_k)}$.

LDA $v'S_w v$ 为按部后散度. (单位 v, 不重要)
(note rank$(S_b) \leqslant C-1$. v 最多取 $C-1$ 个方向 (去分离各类).
and if tr not det
v 不一定是 $S_b v = \lambda S_w v$ 的 eigen解)

CCA
$(X, Y$ 已中心化$)$ $\Sigma_X^{-\frac{1}{2}}\Sigma_{XY}\Sigma_Y^{-\frac{1}{2}}$ $(C < 2$ 时$)$ 奇异decomp.

□ 随机.
$\max \alpha' X' Y \beta$ $(\alpha = \Sigma_{XX}^{-\frac{1}{2}} U_p$, $\beta = \Sigma_{YY}^{-\frac{1}{2}} V_p)$
s.t. $\|X\alpha\|_2 = \|Y\beta\|_2 = 1$.

AB 乘积、低秩近似
CX、CUR
(子空间嵌入) $O(d\log\frac{d}{\varepsilon^2})$. $O(\frac{\log m}{\varepsilon^2})$
长度平方 sampling, random Π proj. ε (离散, J-L Lem.)
杠杆值 for getting 行/列空间主成分. $\|\Pi Ax\|_2^2$ with $\|Ax\|_2^2$.